

When the array to be sorted has length  $n$ , where  $n = 2^k$ , for some nonnegative integer  $k$ , the recurrence relation for the number of comparisons performed by algorithm Mergesort is  $a_n = 2a_{n/2} + n - 1$ , with the initial condition  $a_1 = 0$ . Solving by the telescoping method, we get

$$\begin{aligned}
 a_n &= 2a_{n/2} + n - 1 \\
 &= 2[2a_{n/4} + (n/2) - 1] + n - 1 \\
 &= 2^2 a_{n/2^2} + (n - 2) + (n - 1) \\
 &= 2^2 [2a_{n/2^3} + (n/2^2) - 1] + (n - 2) + (n - 1) \\
 &= 2^3 a_{n/2^3} + (n - 2^2) + (n - 2) + (n - 1) \\
 &\vdots \\
 &= 2^k a_{n/2^k} + (n - 2^{k-1}) + \cdots + (n - 2^0) \\
 &= (2^k)0 + \sum_{i=0}^{k-1} n - 2^i \\
 &= \left( n \sum_{i=0}^{k-1} 1 \right) - \left( \sum_{i=0}^{k-1} 2^i \right) \\
 &= nk - (2^k - 1) \\
 &= n \log_2 n - n + 1 \\
 &= O(n \log n).
 \end{aligned}$$