It is required to derive the number of equivalence relations on the set $X = \{x_1, x_2, \ldots, x_n\}$, that is, the number of ways to partition X into nonempty, disjoint subsets whose union is X. Let this number be a_n.

Suppose that we choose m elements out of $x_1, x_2, \ldots, x_{n-1}$, where $0 \leq m \leq n-1$. The number of equivalence relations that can be defined on these m elements is a_m. To each of these relations, add the $n-m$ leftover elements of X, as an equivalence class by themselves.

Since m elements can be selected out of $n-1$ elements in $\binom{n-1}{m}$ ways, there are $\binom{n-1}{m} a_m$ equivalence relations for a given m.

It follows that

$$a_n = \sum_{m=0}^{n-1} \binom{n-1}{m} a_m,$$

for $n \geq 1$. Note that $a_0 = 1$, since there is one equivalence relation on the empty set, namely, the empty set.

Let us now show that the above algorithm constructs *every* equivalence relation $\mathcal{R}$ on X *exactly once*. Indeed, suppose that the relation $\mathcal{R}$ partitions X into classes $C_1, C_2, \ldots, C_h$. Further, let C_h be the unique class containing x_n. Then $\mathcal{R}$ was constructed uniquely from one of the a_m partitions of the set $C_1 \cup C_2 \cup \cdots \cup C_{h-1}$, where

$$m = \sum_{i=1}^{h-1} |C_i|.$$