The divide-and-conquer algorithm for multiplying two $2n$-bit integers, where $n = 2^k$, $k \geq 0$, executes $f(2n)$ bit operations where

$$f(2n) = 3f(n) + cn,$$

with c a positive integer constant and $f(1) = 1$.

Solving this recurrence by the telescoping method, we obtain:

$$\begin{aligned}
f(2n) &= 3f(n) + cn \\
&= 3\,[3f(n/2) + c(n/2)] + cn \\
&= 3^2 f(n/2) + 3c(n/2) + cn \\
&= 3^2\,[3f(n/4) + c(n/4)] + 3c(n/2) + cn \\
&= 3^3 f(n/4) + 3^2 c(n/4) + 3c(n/2) + cn \\
&\vdots \\
&= 3^{k+1} f(n/2^k) + cn(\sum_{i=1}^{k} 3^i/2^i) + cn \\
&= 3^{k+1} f(1) + cn\left[(\sum_{i=1}^{k} 3^i/2^i) + 1\right] \\
&= (3^{\log_2 2n} \times 1) + cn(\sum_{i=0}^{k} 3^i/2^i) \\
&= (2n)^{\log_2 3} + cn[(3^{k+1}/2^k) - 2] \\
&= (2n)^{\log_2 3} + c3^{k+1} - 2cn \\
&= (2n)^{\log_2 3} + c(2n)^{\log_2 3} - 2cn \\
&= O(n^{1.6}).
\end{aligned}$$

Note that:

1. $\sum_{i=0}^{k} 3^i/2^i$ is a geometric progression whose sum is

$$\frac{\left(\frac{3}{2}\right)^{k+1} - 1}{\frac{3}{2} - 1}.$$

2. $\log_2 3$ equals approximately 1.6.

3. Since $1.6 < 2$, this algorithm is better than standard integer multiplication.