HAND IN
Answers recorded
in question paper

QUEEN'S UNIVERSITY
FACULTY OF ARTS AND SCIENCE
SCHOOL OF COMPUTING

CISC-203*
DISCRETE MATHEMATICS FOR COMPUTING SCIENCE

TEST 2
October 2005

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Please write your answer to each question only in the box marked **Answer**.
No questions will be answered by the instructor during the exam.
This is a closed-book exam. No computers or calculators are allowed.
If you are unsure of what is wanted for a particular question,
make a reasonable assumption and write this at the beginning of your answer.
PLEASE NOTE: Proctors are unable to respond to queries about the interpretation
of exam questions. Do your best to answer exam questions as written.

NAME: ____

STUDENT NUMBER: ________________________

FOR INSTRUCTOR'S USE ONLY

Question 1: ______ / 5

Question 2: ______ / 5

Question 3: ______ / 5

Question 4: ______ / 5

TOTAL: ________ / 20

Question 1: [5 marks]

Let n be a positive integer. Prove that n is even *if and only if* $7n + 4$ is even.

Answer:

First we prove that if n is even then $7n+4$ is even. Since n is even, it can be written as $2k$, for some integer k. Then $7n + 4 = 14k + 4 = 2(7k + 2)$. This is 2 times an integer, which is even.

Now we prove that if $7n + 4$ is even then n is even. Suppose that n is odd; in that case it could be written as $2k + 1$, for some integer k. Thus $7n + 4 = 14k + 11 = 2(7k + 5) + 1$. This is 2 times an integer plus 1, so it is odd. Since this contradicts the given fact that $7n + 4$ is even, our assumption that n is odd must be wrong, and n is even.

Question 2: [5 marks]

Let n be an integer. Prove that n^2 always ends with one of the following digits: 0, 1, 4, 5, 6, or 9.

Hint: Assume without loss of generality that $n \geq 0$, and write $n = 10k + \ell$, where k is a nonnegative integer and $\ell \in \{0, 1, \ldots, 9\}$.

Answer:

There are 10 possible cases for n^2:

$$\begin{aligned}
(10k+0)^2 &= 100k^2 + 0 \\
(10k+1)^2 &= 100k^2 + 20k + 1 \\
(10k+2)^2 &= 100k^2 + 40k + 4 \\
(10k+3)^2 &= 100k^2 + 60k + 9 \\
(10k+4)^2 &= 100k^2 + 80k + 16 \\
(10k+5)^2 &= 100k^2 + 100k + 25 \\
(10k+6)^2 &= 100k^2 + 120k + 36 \\
(10k+7)^2 &= 100k^2 + 140k + 49 \\
(10k+8)^2 &= 100k^2 + 160k + 64 \\
(10k+9)^2 &= 100k^2 + 180k + 81
\end{aligned}$$

In each case, the least significant digit is one of 0, 1, 4, 5, 6, or 9.

Question 3: [5 marks]

Let n be an integer greater than 3. Use mathematical induction to prove that $n^2-7n+12 \geq 0$.

Answer:

Base case: When $n = 4$, $4^2 - 7 \cdot 4 + 12 = 0$.

Inductive assumption: We assume that $n^2 - 7n + 12 \geq 0$.

Inductive step: We must show that $(n+1)^2 - 7(n+1) + 12 \geq 0$.

We have

$$\begin{aligned}(n+1)^2 - 7(n+1) + 12 &= n^2 + 2n + 1 - 7n - 7 + 12 \\ &= (n^2 - 7n + 12) + (2n - 6).\end{aligned}$$

The first parenthesized expression is nonnegative by the inductive assumption, while the second is also nonnegative since $n \geq 4$. Therefore the sum is nonnegative, and the proof is complete.

Question 4: [5 marks]

Give a pseudo-code description of a recursive algorithm for computing the sum of the sequence of numbers $a_1, a_2, \ldots, a_n$.

Answer:

algorithm *sum* $(a_1, a_2, \ldots, a_n)$
if $n = 1$ **then** *sum* $(a_1, a_2, \ldots, a_n) \leftarrow a_1$
else *sum* $(a_1, a_2, \ldots, a_n) \leftarrow a_1 + sum\ (a_2, a_3, \ldots, a_n)$. ■