HAND IN
Answers recorded
in question paper

QUEEN'S UNIVERSITY
FACULTY OF ARTS AND SCIENCE
SCHOOL OF COMPUTING

CISC-203*
DISCRETE MATHEMATICS FOR COMPUTING SCIENCE

TEST 5
December 2005

Professor Selim G. AKL

Please write your answer to each question only in the box marked **Answer**.
No questions will be answered by the instructor during the exam.
This is a closed-book exam. No computers or calculators are allowed.
If you are unsure of what is wanted for a particular question,
make a reasonable assumption and write this at the beginning of your answer.
PLEASE NOTE: Proctors are unable to respond to queries about the interpretation
of exam questions. Do your best to answer exam questions as written.

NAME: ____

STUDENT NUMBER: ____________________________

FOR INSTRUCTOR'S USE ONLY

Question 1: ______ / 5

Question 2: ______ / 5

Question 3: ______ / 5

Question 4: ______ / 5

TOTAL: _________ / 20

Question 1: [5 marks]

(a) Determine whether the relation represented by the following zero-one matrix is an equivalence relation over the set $\{a, b, c, d\}$. Justify your answer.

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Answer:

The relation is reflexive, since all the diagonal elements are 1.
The relation is symmetric, since entry (i, j) is 1 if and only if entry (j, i) is 1.
The relation is transitive, since whenever entries (i, j) and (j, k) are 1, then so is entry (i, k). To see this, recall that a relation R on a set with n elements is transitive if and only if $R^m \subseteq R$ for all $m \leq n$. In our case the adjacency matrix A of R is such that $A = A^2 = A^3 = A^4$, so the relation is clearly transitive.
Therefore, the relation is an equivalence relation.

(b) List the ordered pairs in the equivalence relation produced by the following partition of the set $\{a, b, c, d, e, f, g\}$:

$$\{a, b\}, \{c, d\}, \{e, f, g\}$$

Answer:

$\{(a, a), (a, b), (b, a), (b, b),$
$(c, c), (c, d), (d, c), (d, d),$
$(e, e), (e, f), (e, g), (f, e), (f, f), (f, g), (g, e), (g, f), (g, g)\}$

Question 2: [5 marks]

In the poset $(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |)$

Answer:

(i) Identify the maximal elements
27, 48, 60, 72.

(ii) Identify the minimal elements
2, 9.

(iii) Identify the greatest element, if it exists.
There is no greatest element.

(iv) Identify the least element, if it exists.
There is no least element.

(v) Identify the upper bounds of $\{2, 9\}$.
18, 36, 72.

(vi) Identify the least upper bound of $\{2, 9\}$, if it exists.
18.

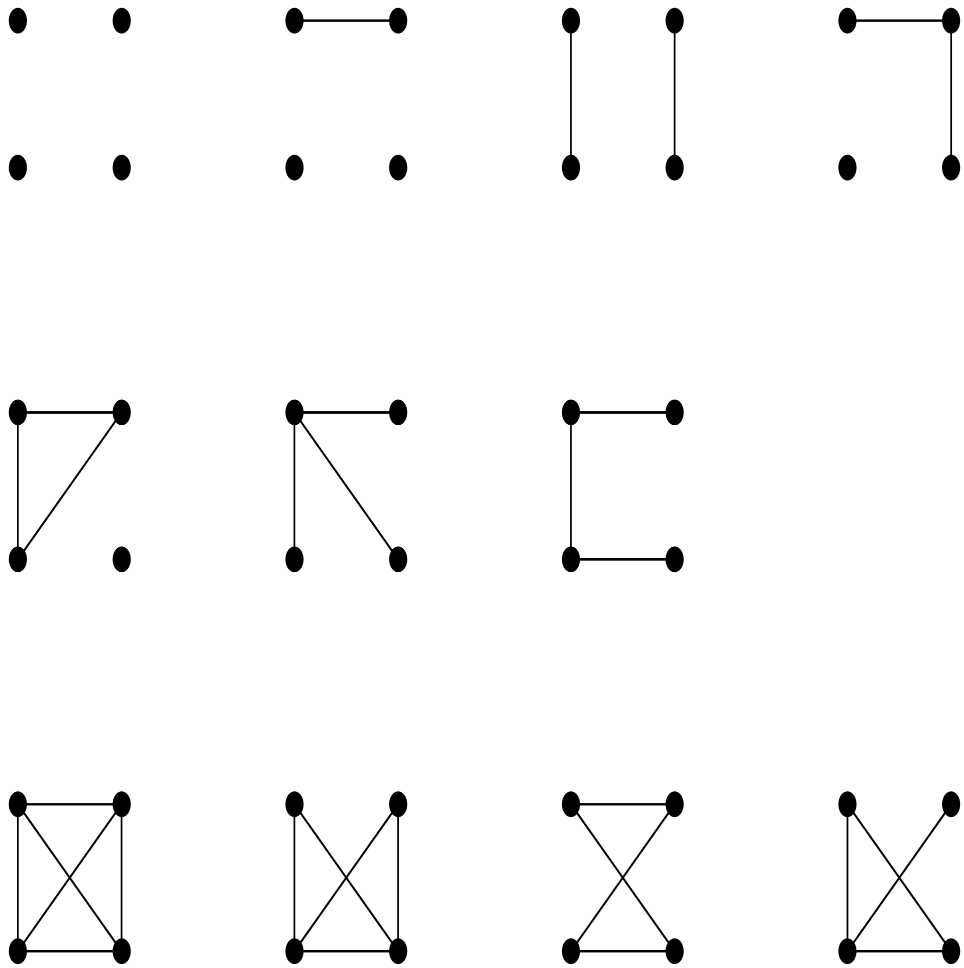
(vii) Identify the lower bounds of $\{60, 72\}$.
2, 4, 6, 12.

(viii) Identify the greatest lower bound of $\{60, 72\}$, if it exists.
12.

Question 3: [5 marks]

Draw all nonisomorphic simple graphs with 4 vertices.

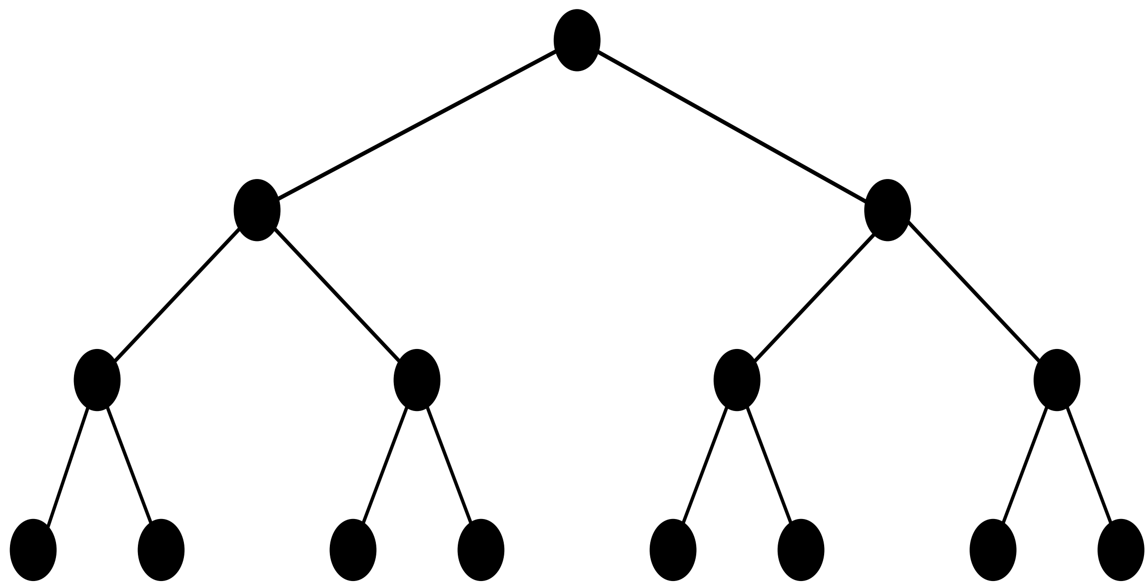
Answer:



Question 4: [5 marks]

(a) Draw a tree-connected network of 15 processors.

Answer:



(b) Describe the fastest way to add the numbers $\{x_1, x_2, \ldots, x_{16}\}$ on the processors of part (a).

Answer:

We first compute $x_1 + x_2, x_3 + x_4, x_5 + x_6, \ldots, x_{15} + x_{16}$ using the 8 processors in the bottom row (the leaves).
These sums are sent up one level, and during the second step we compute $(x_1 + x_2) + (x_3 + x_4)$, $(x_5 + x_6) + (x_7 + x_8)$, ..., $(x_{13} + x_{14}) + (x_{15} + x_{16})$ using the four processors in the third row.
Again, these four sums are sent up to the two processors in the second row, where the sums $(x_1 + x_2 + x_3 + x_4) + (x_5 + x_6 + x_7 + x_8)$ and $(x_9 + x_{10} + x_{11} + x_{12}) + (x_{13} + x_{14} + x_{15} + x_{16})$ are computed.
In the fourth and final step the single processor at the root of the tree computes the final sum by adding $(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8)$ and $(x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16})$.