HAND IN
Answers recorded
in question paper

QUEEN'S UNIVERSITY
FACULTY OF ARTS AND SCIENCE
SCHOOL OF COMPUTING

CISC-203*
DISCRETE MATHEMATICS FOR COMPUTING SCIENCE

TEST 1
October 2005

Professor Selim G. AKL

Please write your answer to each question only in the box marked **Answer**.
No questions will be answered by the instructor during the exam.
This is a closed-book exam. No computers or calculators are allowed.
If you are unsure of what is wanted for a particular question,
make a reasonable assumption and write this at the beginning of your answer.
PLEASE NOTE: Proctors are unable to respond to queries about the interpretation
of exam questions. Do your best to answer exam questions as written.

NAME: ____

STUDENT NUMBER: ___________________________

FOR INSTRUCTOR'S USE ONLY

Question 1: ______ / 5

Question 2: ______ / 5

Question 3: ______ / 5

Question 4: ______ / 5

TOTAL: _________ / 20

Question 1: [5 marks]

(a) Let A and B be two sets. Show that

Answer:

(i) $A = A \cap (A \cup B)$

(ii) $A = A \cup (A \cap B)$

(b) Let A and B be subsets of the finite universal set U. Show that

$$|\overline{A} \cap \overline{B}| = |U| - |A| - |B| + |A \cap B|.$$

Answer:

Question 2: [5 marks]

(a) Determine whether each of the following functions $f\colon \mathbf{Z} \to \mathbf{Z}$ (where $\mathbf{Z}$ is the set of integers) is one-to-one, and justify your answer:

Answer:

(i) $f(n) = n^2 + 1$

(ii) $f(n) = \lceil n/2 \rceil$

(b) Determine whether each of the following functions $f\colon \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$ is onto, and justify your answer:

Answer:

(i) $f(m, n) = 2m - n$

(ii) $f(m, n) = m + n + 1$

Question 3: [5 marks]

(a) Use the Euclidean algorithm to find gcd(2346,5432).

Answer:

(b) Use Fermat's Little Theorem to compute 2^{364} **mod** 7.

Answer:

Question 4: [5 marks]

(a) Compute the Boolean product of the following two matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

Answer:

(b) Let $\mathbf{A} = c\mathbf{I}$, where c is a real number and $\mathbf{I}$ is the $n \times n$ identity matrix, and let $\mathbf{B}$ be an $n \times n$ matrix. Show that $\mathbf{AB} = \mathbf{BA}$.

Answer: