

**HAND IN**  
Answers recorded  
in question paper

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QUEEN'S UNIVERSITY  
FACULTY OF ARTS AND SCIENCE  
SCHOOL OF COMPUTING

CISC-203\*  
DISCRETE MATHEMATICS FOR COMPUTING SCIENCE

TEST 4  
November 2005

Professor Selim G. AKL

Please write your answer to each question only in the box marked **Answer**.

No questions will be answered by the instructor during the exam.

**This is a closed-book exam. No computers or calculators are allowed.**

If you are unsure of what is wanted for a particular question,

make a reasonable assumption and write this at the beginning of your answer.

PLEASE NOTE: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

NAME: \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

FOR INSTRUCTOR'S USE ONLY

Question 1: \_\_\_\_\_ / 5

Question 2: \_\_\_\_\_ / 5

Question 3: \_\_\_\_\_ / 5

Question 4: \_\_\_\_\_ / 5

TOTAL: \_\_\_\_\_ / 20

**Question 1: [5 marks]**

(a) Find a recurrence relation for the number of ways to climb  $n$  stairs,  $n \geq 0$ , if the person climbing the stairs can take one, two, or three stairs at a time.

**Answer:**

(b) What are the initial conditions?

**Answer:**

(c) How many ways can this person climb a flight of eight stairs?

**Answer:**

**Question 2: [5 marks]**

Let  $n = 2^m$ , where  $m = 2^k$ , for  $k \geq 0$ . Now suppose that some function  $f$  satisfies the recurrence relation  $f(n) = 2f(\sqrt{n}) + \log_2 n$ , with  $f(2) = 1$ .

**Answer:**

(i) Compute  $f(16)$ .

**Answer:**

(ii) Solve the recurrence relation for  $f(n)$ .

NAME: \_\_\_\_\_

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**Question 3: [5 marks]**

(a) List all relations on the set  $\{0, 1\}$ .

**Answer:**

(b) Of the relations listed in part (a), identify

**Answer:**

(i) two nonempty reflexive relations

(ii) two nonempty irreflexive relations

(iii) two nonempty symmetric relations

(iv) two nonempty antisymmetric relations

(v) two nonempty asymmetric relations

(vi) two nonempty transitive relations

**Question 4: [5 marks]**

(a) Let  $R$  be a relation on a set of cities  $\{a, b, c, \dots\}$ . The pair  $(a, b)$  is in  $R$  if and only if there is a direct non-stop airline flight from city  $a$  to city  $b$ . When is  $(a, b)$  in

**Answer:**

(i)  $R^2$ ?

(ii)  $R^3$ ?

(iii)  $R^*$ ?

(b) If a relation  $R$  is reflexive, is  $R^*$  necessarily reflexive? Explain your answer.

**Answer:**

(c) If a relation  $R$  is irreflexive, is  $R^2$  necessarily irreflexive? Explain your answer.

**Answer:**