QUEEN'S UNIVERSITY
FACULTY OF ARTS AND SCIENCE
SCHOOL OF COMPUTING

CISC-203*
DISCRETE MATHEMATICS FOR COMPUTING SCIENCE

TEST 1
October 2006

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Please write your answer to each question only in the box marked **Answer**.
No questions will be answered by the instructor during the exam.
This is a closed-book exam. No computers or calculators are allowed.
If you are unsure of what is wanted for a particular question,
make a reasonable assumption and write this at the beginning of your answer.
PLEASE NOTE: Proctors are unable to respond to queries about the interpretation
of exam questions. Do your best to answer exam questions as written.

NAME: _____

STUDENT NUMBER: _____

FOR INSTRUCTOR'S USE ONLY

Question 1: _____ / 5

Question 2: _____ / 5

Question 3: _____ / 5

Question 4: _____ / 5

TOTAL: _____ / 20

Question 1: [5 marks]

Let $A = \{x| -10 \leq x \leq 10\}$, $B = \{x| -15 \leq x \leq 8\}$ and $C = \{x|2 \leq x \leq 15\}$. Let the universe of discourse be $U = Z$, the set of integers. Determine each of the following sets:

Answer:

(a) $(A \cap B) \cup C = \{-10, -9, \ldots, 15\}$

(b) $(A \cup B) \cap C = \{2, 3, \ldots, 10\}$

(c) $(B - C) \cap A = \{-10, -9, \ldots, 1\}$

(d) $(A \cap \bar{B}) \cup C = \{2, 3, \ldots, 15\}$

(e) $(C - \bar{A}) \cup U = U$

Question 2: [5 marks]

(a) Consider the function $f(x) = 20 - 4x^2$ from the set $\{-3, -2, -1, 0, 1, 2, 3\}$ to the set $\{-16, 4, 16, 20, 36\}$. Is it an injection? Is it a surjection? Explain your answer.

Answer:
The function is not an injection. For example, $f(-3) = f(3) = -16$. It is not a surjection because 36 is not an image under f.

(b) Is the function $f(x) = 2x - 1$ a bijection from the set of positive integers to the set of positive integers? Explain your answer.

Answer:
No, because it is not onto. Indeed, none of the even integers is an image under f.

(c) What is the inverse of $f(x) = 5 - 2x^{3/2}$?

Answer:
$f^{-1}(y) = ((5 - y)/2)^{2/3}$

(d) Let $f(x) = x^{2/3} + 2x + 7$ and $g(x) = 3x + 4$ be functions from the set of real numbers to the set of real numbers. What is $f \circ g$?

Answer:
$(3x + 4)^{2/3} + 2(3x + 4) + 7$

(e) For the functions defined in part (d), what is $g \circ f$?

Answer:
$3(x^{2/3} + 2x + 7) + 4$

Question 3: [5 marks]

(a) Use the Euclidean algorithm to find gcd(2468, 8642).

Answer:

$$
\begin{aligned}
8642 &= 2468 \times 3 + 1238 \\
2468 &= 1238 \times 1 + 1230 \\
1238 &= 1230 \times 1 + 8 \\
1230 &= 8 \times 153 + 6 \\
8 &= 6 \times 1 + 2 \\
6 &= 2 \times 3 + 0.
\end{aligned}
$$

gcd(2468, 8642) = 2.

(b) Solve the congruence $3x \equiv 5 \pmod{19}$. Show the steps leading to the solution. Give the answer modulo 19.

Answer:

The inverse of 3 modulo 19 is 13. Multiplying both sides by 13 we get $x \equiv 65 \pmod{19}$. Therefore $x = 8$.

Question 4: [5 marks]

Let

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

Find

(a) $\mathbf{A}^{[2]}$ (recall that $\mathbf{A}^{[2]} = \mathbf{A} \odot \mathbf{A}$, where $\odot$ denotes the Boolean *product* operation)

Answer:

$$\mathbf{A}^{[2]} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(b) $\mathbf{B}^{[2]} \vee \mathbf{A}$ (recall that $\vee$ denotes the Boolean *join* operation)

Answer:

$$\mathbf{B}^{[2]} \vee \mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(c) $\mathbf{A} \wedge \mathbf{B}$ (recall that $\wedge$ denotes the Boolean *meet* operation)

Answer:

$$\mathbf{A} \wedge \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$