HAND IN
Answers recorded
in question paper

QUEEN'S UNIVERSITY
FACULTY OF ARTS AND SCIENCE
SCHOOL OF COMPUTING

CISC-203*
DISCRETE MATHEMATICS FOR COMPUTING SCIENCE

TEST 2
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Please write your answer to each question only in the box marked **Answer**.
No questions will be answered by the instructor during the exam.
This is a closed-book exam. No computers or calculators are allowed.
If you are unsure of what is wanted for a particular question,
make a reasonable assumption and write this at the beginning of your answer.
PLEASE NOTE: Proctors are unable to respond to queries about the interpretation
of exam questions. Do your best to answer exam questions as written.

NAME: ____

STUDENT NUMBER: ____________________________

FOR INSTRUCTOR'S USE ONLY

Question 1: _______ / 5

Question 2: _______ / 5

Question 3: _______ / 5

Question 4: _______ / 5

TOTAL: _________ / 20

Question 1: [5 marks]

Prove that if n is an integer and $3n + 2$ is even, then n is even, **in two different ways of your choice**. Give the name of each proof technique that you use.

Hint: You may use, for example, a direct proof, an indirect proof, a proof by contradiction, and so on.

Answer:

(i) First proof: using a *direct proof*
If $3n + 2$ is even, then so is $3n$. Therefore, $3n = 2k$, for some integer k. It follows that $n = 2(k/3)$, which is even.
(ii) Second proof: using an *indirect proof*
We must prove the contrapositive: If n is odd, then $3n + 2$ is odd.
Assume that n is odd. Then we can write $n = 2k+1$, for some integer k. Then $3n+2 = 3(2k+1)+2 = 6k + 5 = 2(3k + 2) + 1$. Thus $3n + 2$ is odd.
Alternative: using a *proof by contradiction*
Suppose that $3n + 2$ is even, but that n is odd. Since $3n + 2$ is even, so is $3n$. If we add or subtract an odd number from an even number, we get an odd number, so $3n - n = 2n$ is odd. But this is obviously not true. Therefore our assumption was wrong, and n is even.

Question 2: [5 marks]

Let a, b, and c be real numbers. Use a *proof by cases* to show that

$$\min(a, \min(b, c)) = \min(\min(a, b), c).$$

Answer:

There are three cases:

Case 1: a is smallest. That is, $a \leq \min(b, c)$, and the left-hand side equals a. On the right-hand side we have $\min(a, c) = a$ as well.

Case 2: b is smallest. Thus the right-hand side equals $\min(b, c) = b$ and the left-hand side equals $\min(a, b) = b$ as well.

Case 3: c is smallest. Thus the left-hand side is $\min(a, c) = c$, whereas the right-hand side is clearly also c.

Question 3: [5 marks]

Use mathematical induction to prove that

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}.$$

Answer:

The base case, when $n = 1$ reduces to $6 = 6$. Assume the statement is true for some $k \geq 1$. We now prove it for $k + 1$. We have

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + k(k+1)(k+2) + (k+1)(k+2)(k+3)$$

$$= \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3)$$

$$= (k+1)(k+2)(k+3)\left(\frac{k}{4} + 1\right)$$

$$= \frac{(k+1)(k+2)(k+3)(k+4)}{4}.$$

Question 4: [5 marks]

Let n, x, and m be positive integers. Provide a recursive algorithm that computes

$$x^n \textbf{ mod } m$$

using the identity

$$x^n \textbf{ mod } m = (x^{n-1} \textbf{ mod } m \cdot x \textbf{ mod } m) \textbf{ mod } m.$$

Answer:

procedure $power(x, n, m :$ positive integers$)$
if $n = 1$ **then** $power(x, n, m) \leftarrow x \textbf{ mod } m$
else $power(x, n, m) \leftarrow ((x \textbf{ mod } m) \cdot power(x, n-1, m)) \textbf{ mod } m.$

This method, of course, is inefficient.