HAND IN
Answers recorded
in question paper

QUEEN'S UNIVERSITY
FACULTY OF ARTS AND SCIENCE
SCHOOL OF COMPUTING

CISC-203*
DISCRETE MATHEMATICS FOR COMPUTING SCIENCE

TEST 3
November 2006

Professor Selim G. AKL

Please write your answer to each question only in the box marked **Answer**.
No questions will be answered by the instructor during the exam.
This is a closed-book exam. No computers or calculators are allowed.
If you are unsure of what is wanted for a particular question,
make a reasonable assumption and write this at the beginning of your answer.
PLEASE NOTE: Proctors are unable to respond to queries about the interpretation
of exam questions. Do your best to answer exam questions as written.

NAME: ____

STUDENT NUMBER: ______________________________

FOR INSTRUCTOR'S USE ONLY

Question 1: _______ / 5

Question 2: _______ / 5

Question 3: _______ / 5

Question 4: _______ / 5

TOTAL: __________ / 20

Question 1: [5 marks]

Let n be a positive integer, $A = \{1, 2, \ldots, n\}$, and $B = \{x, y, z\}$.

a) How many functions are there from the set A to the set B?

Answer:
There are three possible images in B for each element in A. The total number of functions is therefore 3^n.

b) How many subsets of A have more than one element?

Answer:
The total number of subsets of A is 2^n. Of these, exactly $n + 1$ do not have more than one element, namely the empty set and n subsets with one element each. It follows that $2^n - (n + 1)$ subsets of A have more than one element.

Question 2: [5 marks]

Let n be an integer larger than 1. A computer network consists of n computers. Each computer is directly connected to one or more of the other computers. Show that there are at least two computers that are directly connected to the same number of other computers.

Answer:
The number of connections for each computer is an element of the set $\{1, 2, \ldots, n-1\}$. Since there are n computers, the pigeonhole principle implies that two must have the same number of connections.

Question 3: [5 marks]

An alphabet $\mathcal{A}$ consists of n (distinct) symbols, where $n \geq 5$. Suppose that two of these symbols are x and y. Consider strings of 5 symbols over $\mathcal{A}$, in which no symbol is repeated. How many of these strings contain x immediately followed by y?

Answer:
There are four choices for the position of x in a five-symbol string (since y must follow x). Once the positions of x and y are fixed the other 3 symbols can be selected in $P(n-2,3)$ ways. The total number of strings of 5 symbols in which no symbol is repeated and containing x immediately followed by y is $4P(n-2,3)$.

Question 4: [5 marks]

For a positive integer n, how many different ways are there to seat n people around a circular table, where two seatings are considered the same if one can be obtained from the other by a rotation?

Answer:
Begin by seating one person. There are $(n-1)!$ different ways to seat the others. Since each of these seatings is the same as all of its rotations, the total number of seatings is $(n-1)!$.