HAND IN
Answers recorded
in question paper

QUEEN'S UNIVERSITY
FACULTY OF ARTS AND SCIENCE
SCHOOL OF COMPUTING

CISC-203*
DISCRETE MATHEMATICS FOR COMPUTING SCIENCE

TEST 4
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Please write your answer to each question only in the box marked **Answer**.
No questions will be answered by the instructor during the exam.
This is a closed-book exam. No computers or calculators are allowed.
If you are unsure of what is wanted for a particular question,
make a reasonable assumption and write this at the beginning of your answer.
PLEASE NOTE: Proctors are unable to respond to queries about the interpretation
of exam questions. Do your best to answer exam questions as written.

NAME: ____

STUDENT NUMBER: ____________________________

FOR INSTRUCTOR'S USE ONLY

Question 1: ______ / 5

Question 2: ______ / 5

Question 3: ______ / 5

Question 4: ______ / 5

TOTAL: _________ / 20

Question 1: [5 marks]

(i) Find a recurrence relation for the number of bit strings of length n, $n \geq 0$, that contain at least one substring of three consecutive 0s. Explain your answer.

(ii) What are the initial conditions?

(iii) How many bit strings of length seven contain (at least one substring of) three consecutive 0s?

Answer:
(i) Let a_n be the number of bit strings of length n containing at least one substring of three consecutive 0s. In order to construct one such string, we may start with 1, or 01, or 001 and continue with a string containing three consecutive 0s, or start with 000 and continue with any string. Therefore, for $n \geq 3$, $a_n = a_{n-1} + a_{n-2} + a_{n-3} + 2^{n-3}$.
(ii) $a_0 = a_1 = a_2 = 0$.
(iii) $a_7 = 47$.

Question 2: [5 marks]

Find $f(n)$ when $n = 2^k$, where $k \geq 0$ and $f(n)$ satisfies the recurrence relation $f(n) = f(n/2) + 1$ with $f(1) = 1$.

Answer:
Initially, $f(2^0) = 1$. For each factor of 2 in n, the value of $f(n)$ increases by 1. Indeed, for $k \geq 1$, we have

$$f(2^k) = f(2^{k-1}) + 1 = f(2^{k-2}) + 2 = f(2^{k-3}) + 3 = \ldots = f(2^{k-k}) + k.$$

Therefore, $f(2^k) = 1 + k$.

Question 3: [5 marks]

Let R be the relation

$$\{(1,2),(1,3),(2,3),(2,4),(3,1)\},$$

and let S be the relation

$$\{(2,1),(3,1),(3,2),(4,2)\}.$$

Find $S \circ R$.

Answer:
$S \circ R = \{(1,1),(1,2),(2,1),(2,2)\}$.

Question 4: [5 marks]

Let R be the following relation on the set $\{a, b, c, d, e\}$:

$$\{(a, c), (b, d), (c, a), (d, b), (e, d)\}.$$

Use the 0-1 matrix representation for relations to find the transitive closure of R.

Answer:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad A^{[2]} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

and

$$A^{[3]} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} = A.$$

It follows that $A^{[4]} = A^{[2]}$, and $A^{[5]} = A^{[3]}$. Therefore the transitive closure of R is $A \vee A^{[2]}$, namely,

$$B = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}.$$