CISC 235: Topic 11

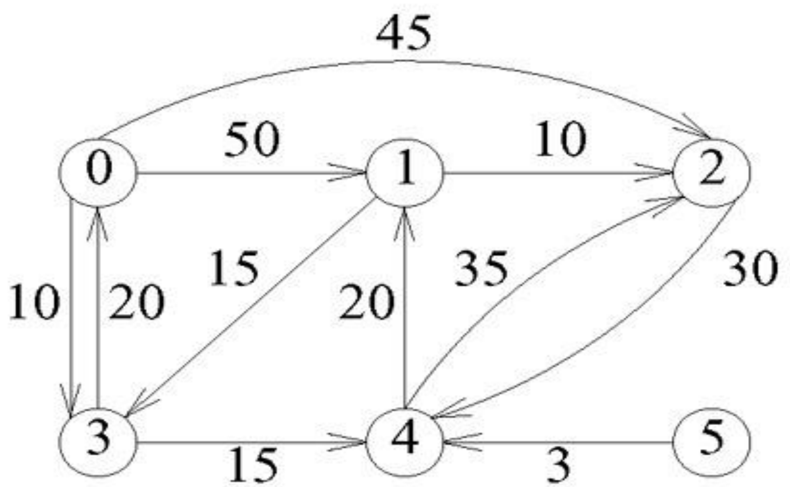
Shortest Paths Algorithms

Outline

- Single-Source Shortest Paths
- Algorithm for Unweighted Graphs
- Algorithm for Weighted, Directed Acyclic Graphs (Weighted DAGs)
- Algorithm for Weighted, Directed Graphs with no negative weights

Single-Source Shortest Paths

Give a graph $G = (V,E)$, we want to find a shortest path from a given source vertex $s \in V$ to each vertex $v \in V$



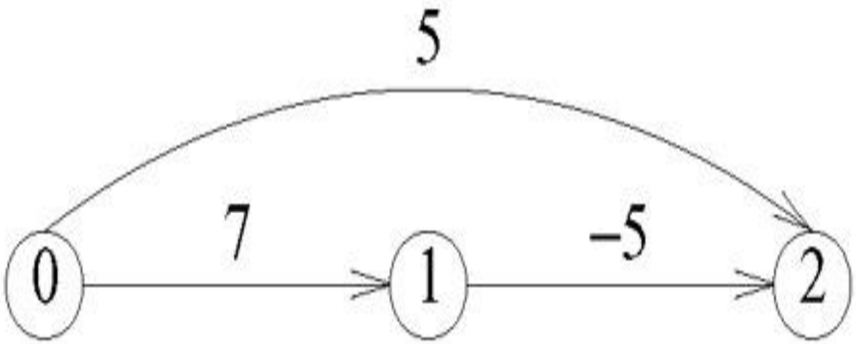
(a) Graph

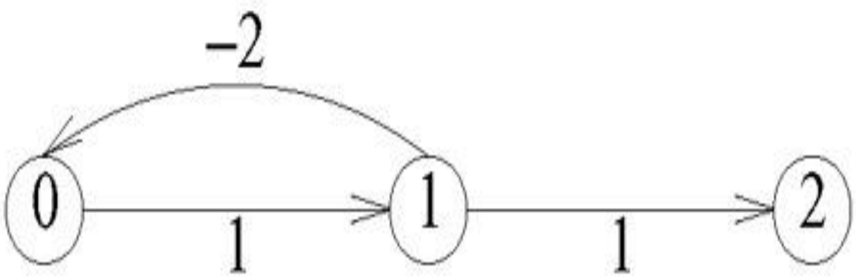
	Path	*Length*
1)	0, 3	10
2)	0, 3, 4	25
3)	0, 3, 4, 1	45
4)	0, 2	45

(b) Shortest paths from 0

Why is vertex 5 not included in the list?

Single-Source Shortest Paths





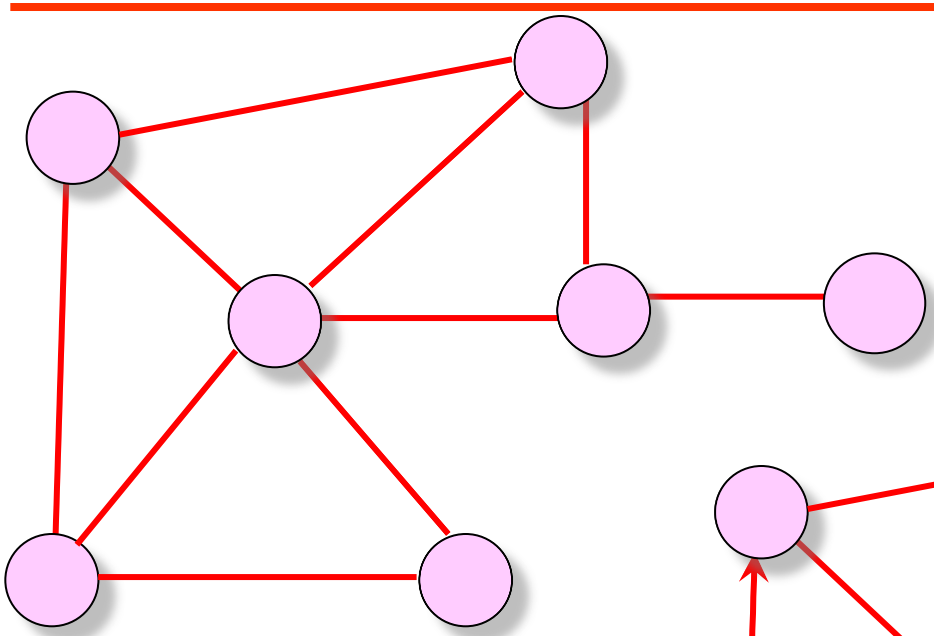
What problems might occur with these two special cases?

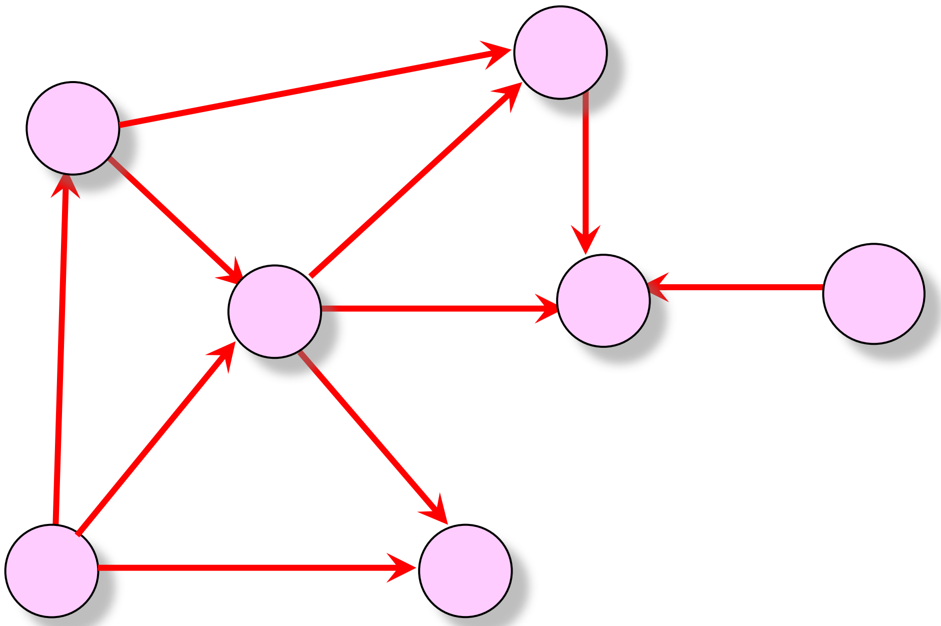
Properties of Shortest Paths

Can a shortest path contain a cycle?

At most how many edges will a shortest path contain?

Unweighted Graphs





What algorithm can be used to find the shortest paths for unweighted graphs?

Shortest Paths Algorithms for Weighted Graphs

For each vertex $v \in V$, we maintain attributes:

d[v] : shortest-path estimate (upper bound on weight of shortest path from source s to v)

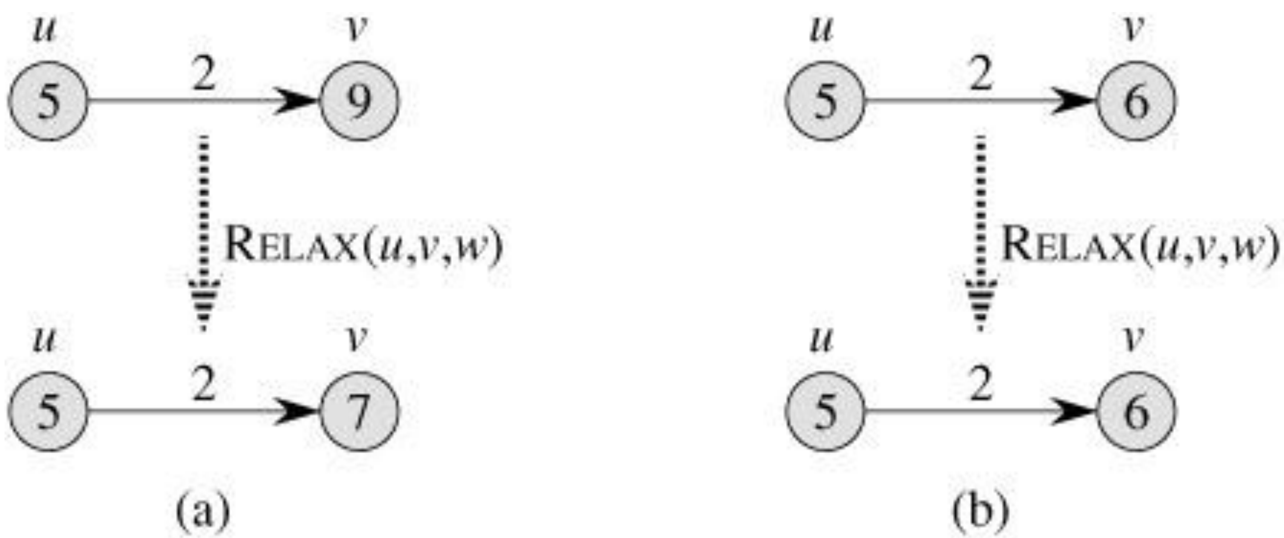
π[v] : predecessor of v on shortest path so far

Initially d[v] is ∞ and π[v] is NIL :

INITIALIZE-SINGLE-SOURCE(G, s)
    for each vertex v ∈ V[G]
        d[v] ← ∞
        π[v] ← NIL
    d[s] ← 0

Updating Adjacent Vertices

RELAX(u, v, w)
        if d[v] > d[u] + w(u, v)
                d[v] ← d[u] + w(u, v)
                π[v] ← u



Algorithm for Weighted DAGs

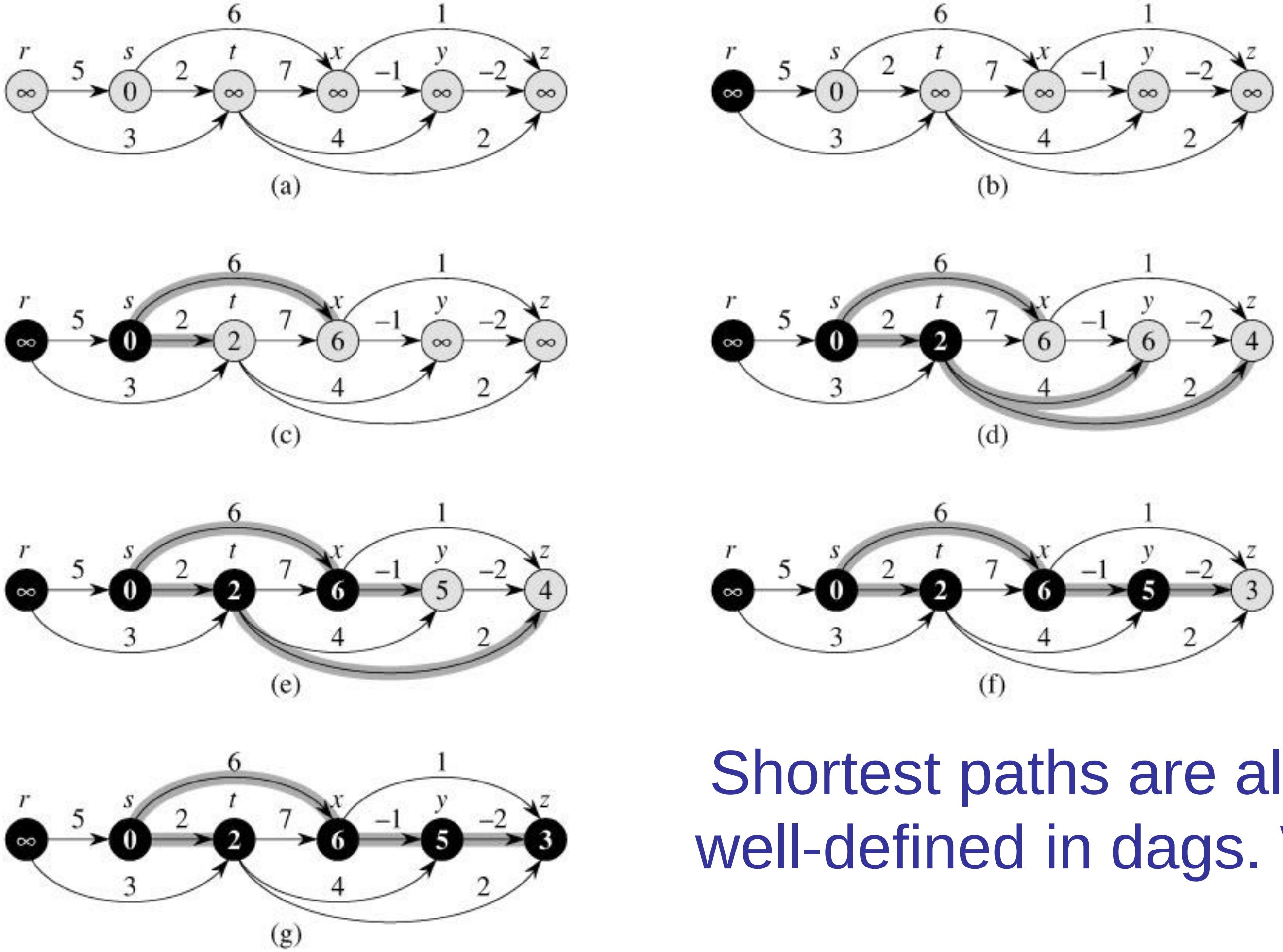
```
AG-SHORTEST-PATHS(G, w, s)

          topologically sort the vertices of G

          INITIALIZE-SINGLE-SOURCE(G, s)

          for each vertex u, taken in topologically sorted order
                for each vertex v ∈ Adj[u]
                      RELAX(u, v, w)
```

Example



Shortest paths are always well-defined in dags. Why?

Weighted Digraphs with No Negative Weights

Dijkstra's Algorithm

DIJKSTRA(G, w, s)

  INITIALIZE-SINGLE-SOURCE(G, s)

  $S \leftarrow \emptyset$

  $Q \leftarrow V[G]$

  while $Q \neq \emptyset$

    $u \leftarrow$ EXTRACT-MIN(Q)

    $S \leftarrow S \cup \{u\}$

    for each vertex $v \in Adj[u]$

      RELAX(u, v, w)

Example