CISC 465*/865
Discrete Mathematics Self-testing Quiz

1. True or false? Give a brief justification of your answer.

   (a) $\emptyset \in \{\{2,1\}, \{1\}, \{2,3\}\}$, where $\emptyset$ denotes the empty set.

   (b) $\emptyset \subseteq \{\{2,1\}, \{1\}, \{2,3\}\}$.

   (c) For all finite sets A and B, $|A \cup B| = |A| + |B|$, where $|A|$ is the number of elements in A.

   (d) If A and B are sets, $A \times B = B \times A$ if and only if $A = B$.

   (e) The following set can be the graph of a function:

   $$\{(0,1), (1,2), (2,3), (3,2), (1,2)\}$$

2. Prove the following for all sets A, B, and C:

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

3. Prove that if R is an equivalence relation on a set A, then so is the inverse relation R^{-1}.

4. Prove that if the composition of $f: A \to B$ and $g: B \to A$ is the identity function on A, then f is one-to-one (injective) and g is onto (surjective).

References

K. Devlin. *Sets, Functions, and Logic: An Introduction to Abstract Mathematics*. Chapman and Hall, London, 2nd edition, 1992.

D. Velleman. *How to Prove it: a Structured Approach*. Cambridge University Press, 1994.