

CISC235  
Winter 2007  
Homework for week 11  
in preparation for quiz 5

Use the definition of a B-Tree given below for the next two questions.

A B-tree of order  $M$  is a  $M$ -ary tree,  $M \geq 2$  with the following properties:

- The root has at least two subtrees unless it's a leaf.
- Each non-root holds  $k-1$  keys, and each non-root and non-leaf also holds  $k$  pointers to subtrees, where the ceiling of  $m/2 \leq k \leq m$ .
- All leaves are on the same level.
- The keys in each node are in non-decreasing order.
- The children are in order: the keys in the first  $i$  children are less than or equal to the  $i$ th key the keys in the last  $m-i$  children are greater than or equal to the  $i$ th key

1. What is the maximum number of keys that a B-Tree of order 6 (at most five keys per node) and of height 4 hold? How many disk accesses in the worst case would be required to find any key, assuming one block on disk can hold one node? How many keys can a B-Tree of order  $m$  and of height  $h$  hold?

### External Merge Sorting

Suppose that there are 32 records to be sorted and the data is stored on external memory. Using an I/O model where reading or writing one record counts as one external memory access answer the following questions.

2. How many external memory accesses (reads and writes) are used to sort the data using a 2-way merge sort? Assume that all you can do in internal memory is compare two numbers and output the smallest.
3. How many external memory accesses (reads and writes) are used to sort the data using a 4-way merge sort? Assume that all you can do in internal memory is compare four numbers and output the smallest.
4. What if we have enough internal memory to make initial runs of 8 sorted records. How many external memory accesses (reads and writes) are used to sort the data using a 2-way merge sort? 4-way merge sort?
5. Illustrate how the 2-way and 4-way merge sorts of part c) would work using the following set of random integers.  
 41 40 35 6 74 8 87 89 99 24 2 8 44 19 93 23  
 63 91 4 5 70 57 38 59 3 4 83 83 46 18 53 17
6. Give one possible insertion sequence that leads to the following skip list.
7. What is the average number of comparisons needed for a successful search in the skip list shown.

