CISC271
Fall 2006
Homework for week 5
in preparation for quiz 2
Solutions

This homework is about norms and condition numbers.

Recktenwald Chapter 8. questions 22, and 35.

8-22 Solution: I wrote a function that solves the linear system of equations and does the plotting. Here it is:

```
function c = ParabolaDR(exes,wys)
%Answers question 22 in Chapter 8. of Recktenwald.
V = vander(exes);
c = V \ wys;
X = linspace(-10,10,100);

for k = 1:100
%evaluate using Horner's method
Y(k) = c(1);
for i = 2:3
   Y(k) =  Y(k)*X(k)  + c(i);
end
end

%Y = polyval(c,X);  This is the easy way to evaluate a polynomial in Matlab
plot(X,Y, exes,wys, '+');
```

Note that I used the Matlab operation `vander` to automatically set up a Vandermonde matrix. Here's the output from `help vander`.

```
VANDER Vandermonde matrix.
    A = VANDER(V) returns the Vandermonde matrix whose columns
    are powers of the vector V, that is A(i,j) = v(i)^(n-j).
```

For question 22 determine the condition numbers of the matrices used to obtain the interpolating parabola.

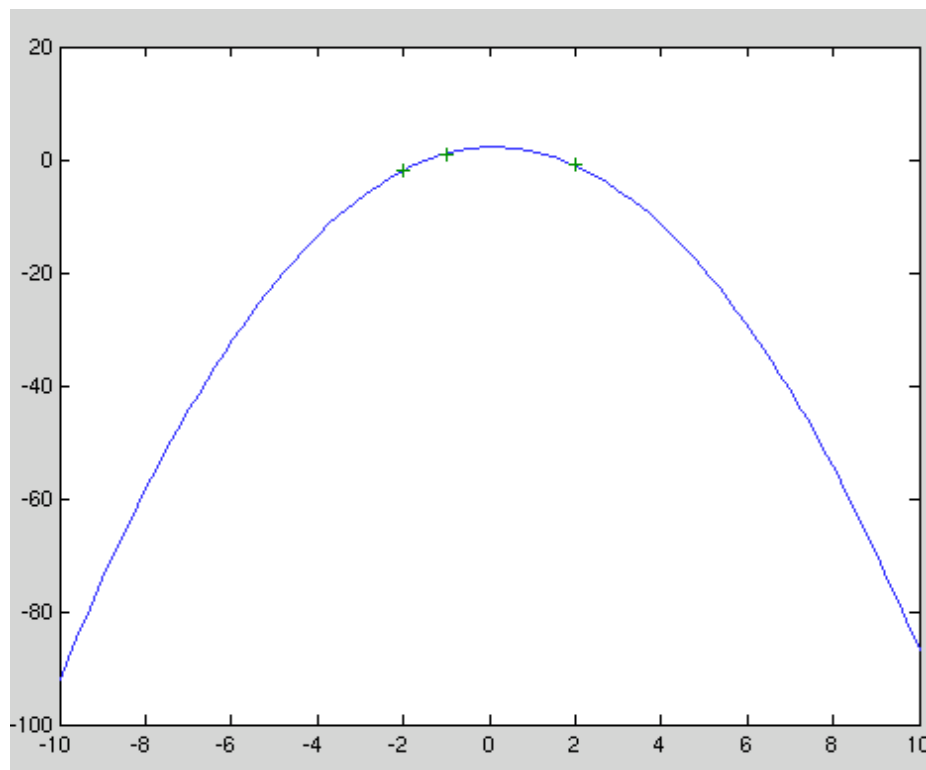It's easy enough to do as follows:

```
V = vander(exes)
C = cond(V)

V =

        4      -2       1
        0       0       1
        4       2       1

C =

    6.0211
```

I plotted all of the Parabolas. The one in (d) is a line, and (e) yields a singular system. Here is the prettiest. It's (c).

None of the condition numbers, except for the singular system, were very big due to the small problem size and small numbers. For the singular system the condition number of the Vandermonde matrix is 8.8289e+17.
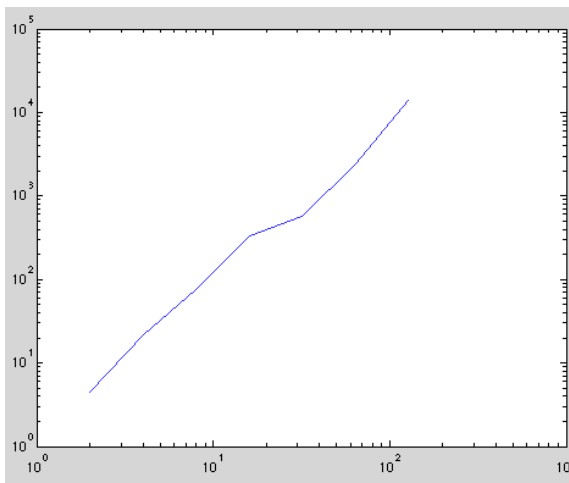
8-35 Solution: My matlab M-file:

```
function condsurveyDR
%Answers question 35 in Chapter 8. of REcktenwald.

for i = 1:7
    ave = 0; miny = realmax; maxy = 1;
    X(i) = 2^(i);
    for j = 1:12
      A = rand(X(i));
      C = cond(A,2);
      miny = min(C,miny);
      maxy = max(C,maxy);
      ave = ave + C/10;
    end
    %Throw out min and max value in an effort to smooth things out
    ave = ave - (miny/10 + maxy/10);
    Aave(i) = ave;
end

loglog(X,Aave);
```

And this is the plot I obtained:

Note: I use 12 samples and throw out the max and min. This seems to smooth things out a bit. For question 35 repeat the experiments using $\mathcal{K}_1(A)$ and $\mathcal{K}_\infty(A)$. Similar results are observed using $\mathcal{K}_1(A)$ and $\mathcal{K}_\infty(A)$. There is an increase in condition number proportional to the size of the random array.

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