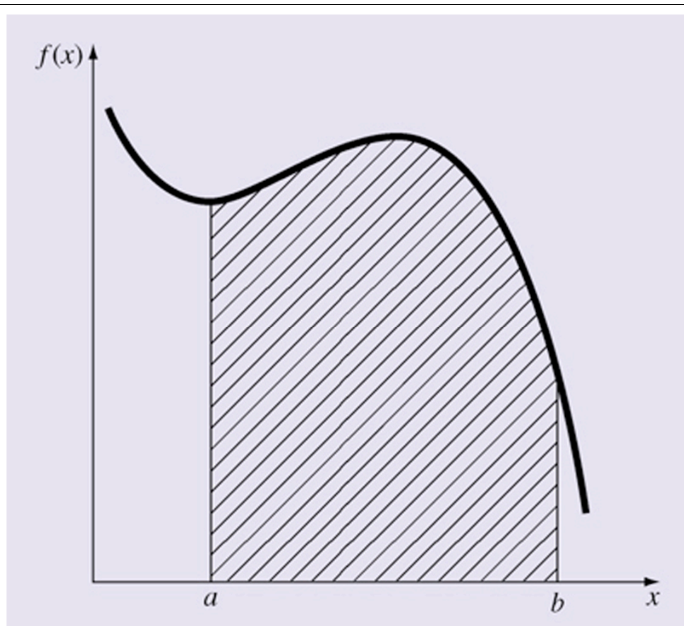
# Numerical Integration

## Motivation

Material shown on the following pages has been adapted from:

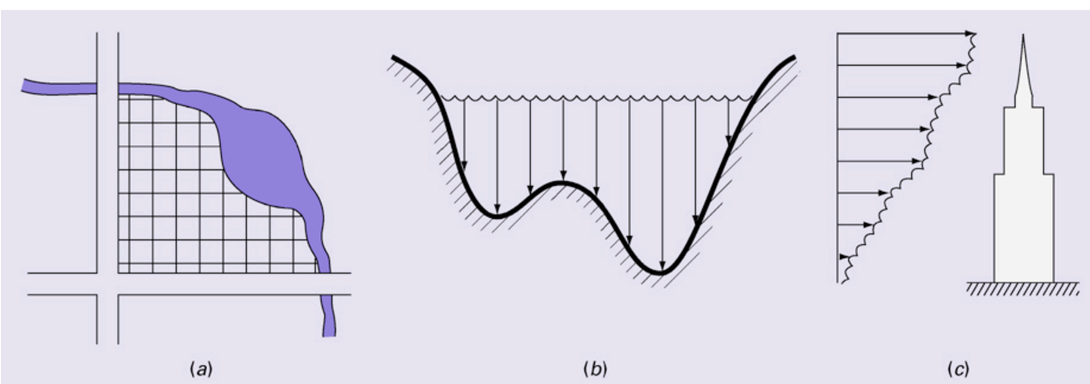
- **Applied Numerical Methods** *With MATLAB® for Engineers and Scientists* **by Steven C. Chapra**
- The Wolfram MathWorld web site: http://mathworld.wolfram.com/NormalDistribution.html



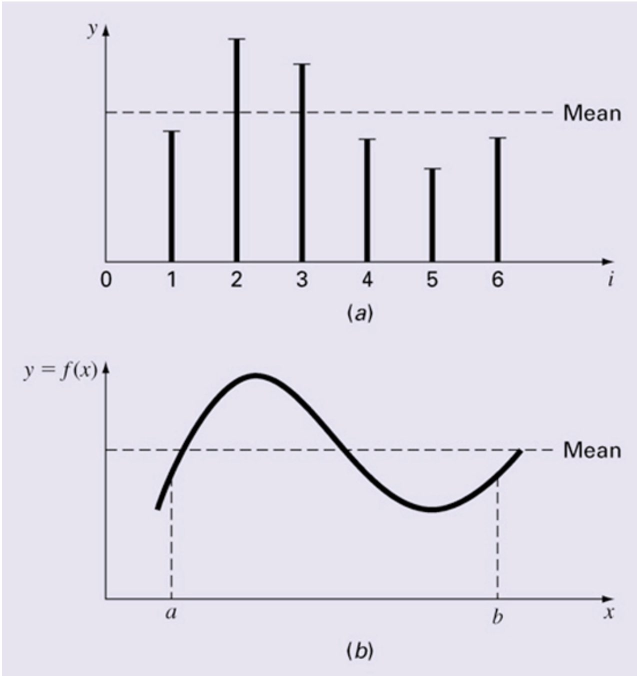

$\int_a^b f(x)dx$ represents an area under a curve.

- In a calculus course we learn how to integrate analytically.
- In some cases we get sampled data for which we need to compute an integral, thus numerical techniques are necessary.
- In other cases there are no known analytic techniques for obtaining the integral.
- A few illustrative examples are given.

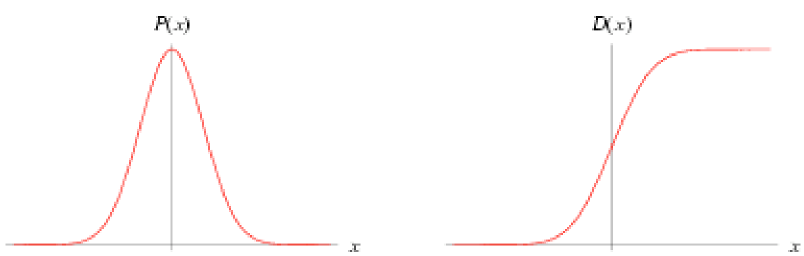
# Sampled Data Examples




• A surveyor might need to know the area of a field bounded by a meandering stream and two roads.
• A hydrologist might need to know the cross-sectional area of a river.
• A structural engineer might need to determine the net force due to a nonuniform wind blowing against the side of a skyscraper.




In statistics the mean of a continuous distribution is obtained by integrating.




$$\Phi(z) \equiv \frac{1}{\sqrt{2\pi}} \int_0^z e^{-x^2/2}\, dx = \frac{1}{2} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right),$$

The normal distribution function ϕ(Z) gives the probability that a standard normal variate assumes a value in the interval [0,z], where erf is a function sometimes called the error function. Neither ϕ(Z) nor erf can be expressed in terms of finite additions, subtractions, multiplications, and root extractions, and so both must be either **computed numerically** or otherwise approximated.