CISC271
Fall 2006
Homework for week 8
in preparation for quiz 4
Solutions

This homework is about numerical quadrature..

Q1. I derived the relation E2 = (S2 -S1)/3 for the adaptive trapezoid rule. Do a similar derivation for Simpson's rule.

Solution: I actually did this in class also. For Simpson's rule we have the error term as roughly ch^5, where c can be considered to be a constant over a small interval. Using the notation as in class we have

$$S1 + E1 = S2 + E2 \tag{1}$$

Thus E2 $\approx$ E1/16 so using equation (1) we have S1 + E1 $\approx$ S2 + E1/16 which implies E2 $\approx$ (S1-S2)/15.

Q2. What do the following lines of Matlab do? More to the point what is the value of Z?

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X = linspace(0,1)
Y = X.^2
Z = (sum(Y) - 0.5)/99
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Solution: This works as a method that uses the trapezoid rule to approximate the integral

$$\int_0^1 x^2 dx.$$

There are 99 panels and the width of each panel is 1/99. We have to divide Y(1) and Y(100) by 2. Since Y(1) 0^2 there is no need to divide it. Y(100) = 1 therefore Y(100)/2 = 0.5, explaining why I subtract 0.5 from sum(Y).

The trapezoid rule on 99 panels yields 0.3334, and the actual area is 1/3 so the approximation we got is reasonably good.

The following questions are from Recktenwald Chapter 11.

Questions 1 and 5. asks you to evaluate two integrals by hand and then use the trapezoid (Simpson's rule) by hand and then use the supplied trapezoid (simpson) routines. If you have access to the symbolic toolbox (this should be installed on the school's machines and it comes with the student version of Matlab) try integrating using the built in symbolic integrator *int*.

Solution: My intent was for you to apply the trapezoid and Simpson's rules to the integrals of question 1. At least one of you thought that I meant you to do questions 4 and 5 and to use the integrals of question 4. I will solve both here.

Integrating exactly, I obtained the values:

$$\int (x^2 + x + 1)dx = \frac{x^3}{3} + \frac{x^2}{2} + x$$

and

$$\int (x^3 - 1)dx = \frac{x^4}{4} - x.$$

Thus the definite integrals are:

$$I_1 = \int_{-\pi/2}^{\pi} (x^2 + x + 1)dx = 20.0408$$

and

$$I_2 = \int_{\sqrt{(3)}}^{-5} (x^3 - 1)dx = 160.7321.$$

Then using the trapezoid and simpson functions I obtained the following approximations.

Two panel trapezoid A1 = 24.4011 and A2 = 223.0477 Two panel simpson A1 = 20.0408 and A2 = 160.7321

Note: A one panel application of Simpson's rule would have obtained the same value for A1 and A2.

For the functions in question 4 we have

$$\int xe^{-x}dx = -xe^{-x} - e^{-x}$$

and

$$\int \cos(x)dx = sin(x).$$

Thus the definite integrals are:

$$I_1 = \int_0^2 xe^{-x}dx = -3e^{-2} + 1 \approx 0.594$$

and

$$I_2 = \int_0^\pi cos(x)dx = 0.$$

Then using the trapezoid and Simpson's method I obtained the following approximations.

Two panel trapezoid A1 = 0.5032 and A2 = 1.7439e-16 $\approx$ 0 Two panel Simpson's A1 = 0.5930 and 1.7439e-16 $\approx$ 0

Note: With this example a one panel application of the trapezoid rule gives the exact answer!

Questions 16 and 25 asks you to compare the performance of different numerical quadrature functions. I used the gaussquad function with 2 nodes per panel to make the comparisons more interesting. The default for gaussquad is 4 nodes per panel. If run with 4 nodes per panel gaussqaud appears to be 1000 times more accurate than the other two methods.

Solution:

$$I = \int_0^1 (\sqrt{x})dx = 2/3$$

and my experimental results are:

	2	4	8	16
trapezoid	0.6036	0.6433	0.6581	0.6636
simpson	0.6565	0.6631	0.6654	0.6662
gaussquad	0.6692	0.6676	0.6670	0.6668

Here is the same table with absolute errors instead of actual areas.

	2	4	8	16
trapezoid	0.0631	0.0234	0.0085	0.0031
simpson	0.0101	0.0036	0.0013	0.0004
gaussquad	0.0026	0.0009	0.0003	0.0001

Now using the adaptive algorithms

	0.1	0.01	0.001	0.0001
adaptsimpson	0.6565	0.6565	0.6631	0.6667
quad	0.6654	0.6654	0.6654	0.6662
quadl	0.6650	0.6650	0.6666	0.6666

and the absolute errors as a function of tolerance

	0.1	0.01	0.001	0.0001
adaptsimpson	0.0101	0.0101	0.0036	0.0000
quad	0.0013	0.0013	0.0013	0.0004
quadl	0.0017	0.0017	0.0000	0.0000