Differential Equations

- Ubiquitous in science and engineering.
- Used to model rates of change.
- Analytic solutions are often unknown.
- Even if an analytic solution is known, evaluating that function may be computationally costly.
- Numerical methods for solving differential equations are commonly used.

Differential Equations

- We will be studying a special class of differential equations the so called *ordinary differential equations* (ODE).
- All the differentials in an ordinary differential equation are differentials with respect to the same variable, i.e. an ODE does not contain any partial differentials.

Radioactive Decay

Radioactive decay is the set of various processes by which unstable atomic nuclei emit subatomic particles (radiation).

The decay of an unstable nucleus is entirely random and it is impossible to predict when a particular atom will decay. However, it is equally likely to decay at any time.

Radioactive Decay

If the population of these nuclei at time *t* is represented as the function *y(t)*, then *y(t)* corresponds to the differential equation:

$$\frac{dy(t)}{dt} = -Ky(t)$$

where *K* is the <u>decay constant</u>, or the proportionality constant between the rate of radioactive, nuclear decay and the number of radioactive nuclei remaining.

Radioactive Decay

The radioactive *half-life* for a given radioisotope is a measure of the tendency of the nucleus to "decay" or "disintegrate" and as such is based purely upon that probability. The decay constant K is proportional to the half life $t_{1/2}$.

$$K = \frac{ln(2)}{t_{1/2}}$$

Radioactive Decay

The population of these nuclei at time t is represented as the function $y(t)$ (or $y = g(t)$). Physics gives us a recipe for the first derivative y' as follows:

$$\frac{dy(t)}{dt} = -Ky(t)$$

where K is the decay constant, and an initial condition $y(0)$, i.e the number of nucleii at time 0. The solution to this initial value problem, is well known, and is:

$$y(t) = y(0)e^{-Kt}$$

Radioactive Decay

For example the half life of radium is 1602 years. It decays into Radon gas. Using this value we obtain a decay constant for radium of 4.3268e-04.

Starting with a kilogram of Radon at time $t = 0$, after one year we expect tha

$$y(1) = e^{-4.3268e-04}$$

or 0.9996 kg.

Radioactive Decay

For example the half life of radium is 1602 years. It decays into Radon gas. Using this value we obtain a decay constant for radium of 4.3268e-04.

We can use Matlab to obtain a numerical solution to this initial value problem:

```
> diffeq = inline('-4.3268e-04*y','t','y');

> ode45(diffeq,1,1);

ans =

    0.9996
```