CISC271
Fall 2006
Homework for week 10
in preparation for quiz 5
Solutions

The following questions are from Recktenwald Chapter 12.

12-3 We have $y_1 = y_0 + hf(t_0, y_0)$ or $y_1 = 0 + (0.2)(\frac{1}{0+0+1}) = 0.2$ Similarly $y_2 = 0.2 + (0.2)(\frac{1}{0.2+0.2+1}) = 0.3429$ and $y_3 = 0.3429 + (0.2)(\frac{1}{0.4+0.3429+1}) = 0.4576$

12-4 I used the following Matlab instructions:

```
syms y t
diffeq = @(t,y) 1/(t + y + 1);
[a, b]  = odeEuler(diffeq,1,.2,0)
```

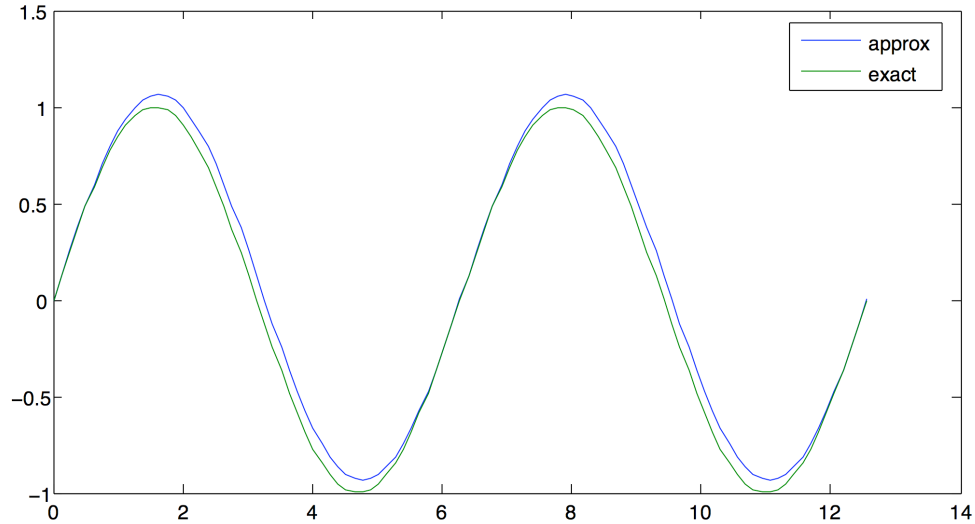
And I got the following output:

```
a =   0
    0.2000
    0.4000
    0.6000
    0.8000
    1.0000

b =  0
    0.2000
    0.3429
    0.4576
    0.5548
    0.6397
```

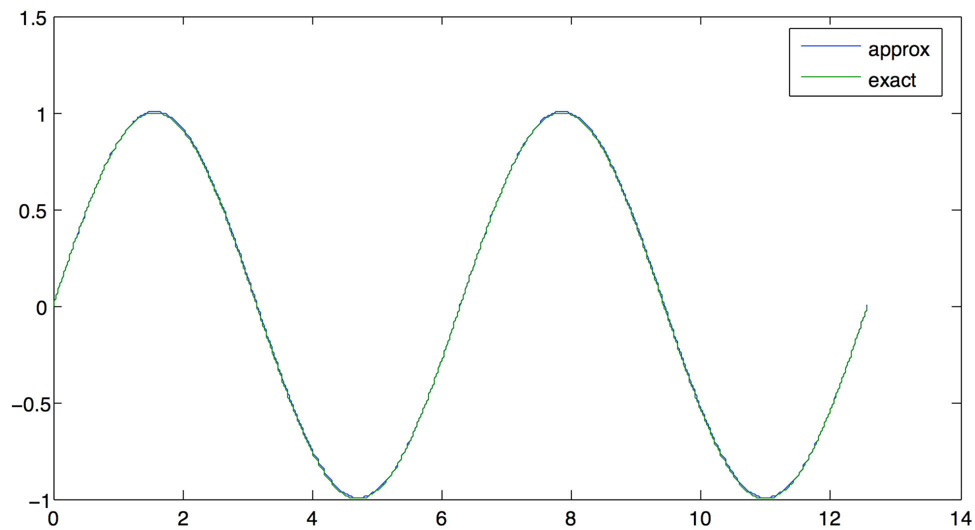
12-5 Here's my m-file:

```
function demoEulerQ5DR
% demoEulerQ5DR  Integrate dy/dt = cos(t);  y(0) = 0 with Euler's method
% for 0 <= t <= 4pi
%
% Synopsis:  demoEulerQ5DR
% Input:    none
% Output:    A plot comparing the numerical and exact solutions for two
% values of step size h

syms tt yy;
rhs = @(tt,yy) cos(tt);

tn = 4*pi;  y0 = 0;  %  stopping time and IC

h = [4*pi/100 4*pi/1000];
maxtn = [ 0 0 ];
maxabserr = [ 0 0 ];

for i = 1:2
      [t,y] = odeEuler(rhs,tn,h(i),y0);     %  Euler integration with t0 = 0
      yex = sin(t);    %  Exact solution for all values of t.
      maxtn(i) =  abs(y(length(t))-0);
      maxabserr(i) = norm(y-yex,inf);

      % --- Plot the results
      subplot(2,1,i);   plot(t,y,t,yex);
      legend('approx','exact');
      text(1,-1.5,sprintf('Max err at 4\\pi = %8.2e  max abs err = %8.2e'
         'for h = %f', maxtn(i),maxabserr(i),h(i)));
end
```

And here are my plots and error reports:



Max err at 4π = 9.58e−16 max abs err = 1.26e−01 for h = 0.125664



Max err at 4π = 9.94e−16 max abs err = 1.26e−02 for h = 0.012566

12-6 Here's my m-file:

```
function demoEulerQ6DR
% Integrate dy/dt = cos(t);  y(0) = 0 with Euler's method for 0 <= t <= 4pi
%
% Synopsis:  demoEulerQ5DR
% Input:    none
% Output:    A plot comparing the numerical and exact solutions for four
% values of step size h
syms tt yy; rhs = @(tt,yy) cos(tt);
```

```
tn = 4*pi;  y0 = 0;  %  stopping time and IC

h = [4*pi/10 4*pi/100 4*pi/1000 4*pi/10000 ];
maxtn = [ 0 0 0 0 ]; maxabserr = [ 0 0 0 0 ];

for i = 1:4
          [t,y] = odeEuler(rhs,tn,h(i),y0);     %  Euler integration with t0 = 0
         yex = sin(t);    %  Exact solution for all values of t.
         maxtn(i) =  abs(y(length(t))-yex(length(t)));
        maxabserr(i) = norm(y-yex,inf);
end

% --- log log Plot the results
loglog(h,maxtn,'bo',h, maxtn,'b',h,maxabserr,'ro',h, maxabserr,'r');
legend('max err','max err','max abs err','max abs err','Location','SouthOutside');
```

My plots show that although the maximum absolute error decreases with the value of h, the error at t_n is relatively flat. Observe that $y(t_n) = y(0)$ and we have a periodic function. This may explain why the error at t_n is negligible. The size of h cannot be decreased indefinitely, as there is a a floating point realmin lower bound. Presumably arithmetic error would dominate any improved performance well before the step size reaches realmin.