CISC271

Practice Final Exam

2005

These questions should give you some idea of the *type* of question I may ask on the final.

1. I have reproduced the definitions of the MATLAB constants realmin and realmax and eps.

   Built-in Variable: realmin The smallest normalized floating point number that is representable. The actual value is system-dependent. On machines that support 64-bit IEEE floating point arithmetic, 'realmin' is approximately 2.2251e-308

   - Built-in Variable: realmax The largest floating point number that is representable. The actual value is system-dependent. On machines that support 64-bit IEEE floating point arithmetic, 'realmax' is approximately 1.7977e+308

   Built-in Variable: eps The machine precision. More precisely, 'eps' is the largest relative spacing between any two adjacent numbers in the machine's floating point system. This number is obviously system-dependent. On machines that support 64 bit IEEE floating point arithmetic, 'eps' is approximately 2.2204e-16.

   (a) Is (realmin + eps) - realmin equal to zero? Explain.

   (b) Is (realmax - eps) - realmax equal to zero? Explain.

2. The secant method for determining the root of a function can be implemented with the following formula:

$$x_{k+1} = x_k - f(x_k)\left[\frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}\right]$$

   Explain how this formula may produce results that are completely unusable. (HINT: What happens to the denominator when $f(x_k)$ is approximately equal to $f(x_{k-1})$?)

3. Using the secant algorithm and with initial values $a = 4$ and $b = 6$ what would be the next value obtained using the secant algorithm for the function:

$$x^2 - 25 = 0$$

   Show your work.

4. Suppose that you are given the following experimental data which are from a function f(x):

x_i	1	2	3
y_i	2	4	2

Using Lagrange's formula find the unique polynomial that passes through all the points.

5. (4) Consider the following matrix.

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

(a) Perform the calculations for Gaussian elimination by hand, to put the matrix B in triangular form. Show all of your work.

(b) What is the LU decomposition of B.

(c) What would the Matlab be to obtain an LU decomposition of B.

(d) Using the L_∞ matrix norm calculate the condition number of B.

6. Use Simpson's Rule to numerically estimate

$$\int_1^{2.718} \frac{1}{x} dx$$

The answer correct to 4 decimal places is 0.9999. What is the relative error of your computation.

7. Evaluate $I = \int_0^1 x^2$ by a four point Guassian Quadrature formula. Just write out the formula, in terms of the points x_1, x_2, x_3, x_4 and weights w_1, w_2, w_3, w_4.

8. This question deals with least-squares approximations. Suppose that you were given many data points (e.g., a hundred) and that polynomials did not fit well. Plotting the data, you suspected that the y_i values were exponentially related to the x_i values. Show how'to fit data to the function

$$y = ce^{kx}$$

that is, estimate values for c and k in the least squares sense.