CISC271
Fall 2005
Homework for week 8
in preparation for quiz 3
Solutions

This homework is about piece-wise interpolation.

Recktenwald Chapter 10. questions 29, 31, 32

For question 32 use the Matlab spline with the not-a-knot (default) end condition as well as the so called "fixed slope" end condition, use $y' = 0$ at the ends. Use `help spline` to see how to do this.

8-29 Recall we have

$$P_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

$$P_i'(x) = b_i + 2c_i(x - x_i) + 3d_i(x - x_i)^2$$

Let $h_i = x_i - x_{i-1}$. We will use the equations (10.39) (page 539 of Recktenwald) for a_i, b_i, c_i, and d_i.

$$a_{i-1} = y_{i-1} \quad (1)$$

$$b_{i-1} = b_{i-1} \quad (2)$$

$$c_{i-1} = \frac{3f[x_{i-1}, x_i] - 2b_{i-1} - bi}{h_i} \quad (3)$$

$$d_{i-1} = \frac{b_{i-1} - 2f[x_{i-1}, x_i] + b_i}{h_i^2} \quad (4)$$

Thus $P_i(x_i) = y_i = a_i$ and $P_i'(x_i) = b_i$. It remains to show that $P_{i-1}(x_i) = a_i$ and that $P_{i-1}'(x_i) = b_i$.

Thus,

$$\begin{aligned} P_{i-1}(x_i) &= y_{i-1} + b_{i-1}h_i + c_{i-1}h_i^2 + d_{i-1}h_i^3 \\ &= y_{i-1} + b_{i-1}h_i + h_i(3f[x_{i-1}, x_i] - 2b_{i-1} - b_i) + h_i(b_{i-1} - 2f[x_{i-1}, x_i] + b_i) \\ &= y_{i-1} + h_i f[x_{i-1}, x_i] \\ &= y_{i-1} + h_i(y_i - y_{i-1})/h_i \\ &= y_i \end{aligned}$$

And,

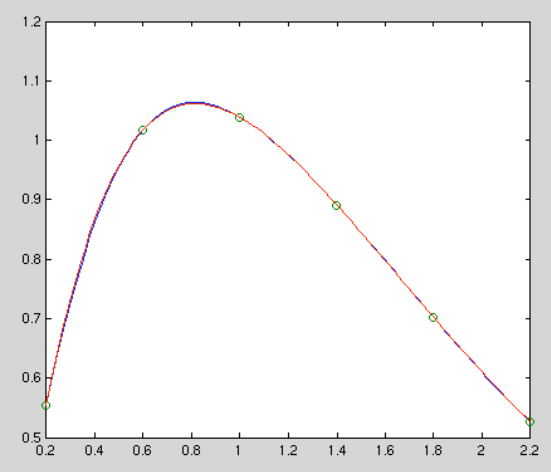
$$\begin{aligned} P'_{i-1}(x_i) &= b_{i-1} + 2c_{i-1}h_i + 3d_{i-1}h_i^2 \\ &= b_{i-1} + 2(3f[x_{i-1}, x_i] - 2b_{i-1} - b_i) + 3(b_{i-1} - 2f[x_{i-1}, x_i] + b_i) \\ &= b_i \end{aligned}$$

10-31 No it isn't, because one needs to evaluate the entire system of linear equations to determine any of the coefficients.

10-32 Here are the Matlab instructions that I used.

```
> xx = linspace(0.2,2.2);
> yy = spline(x,y,xx);
> plot(xx,yy,x,y,'o',xx, sqrt(12.5) .* xx .* exp(-sqrt(1.5).*xx))
```

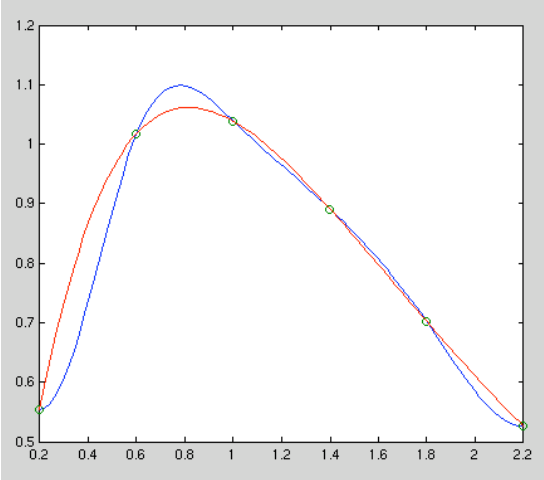
And here is the plot that I obtained.



For the "fixed slope" slope = 0 end conditions I used

```
yy = spline(x,[0 y 0],xx);
```

The plot did not look as nice. Here it is:


1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2