CISC271
Fall 2005
Homework for week 6
in preparation for quiz 2
Solutions

This homework is about norms and condition numbers.

Recktenwald Chapter 8. questions 22, 30, 31, and 35.

8-22 I wrote a function that solves the Vandermonde system and does the plotting. Here it is:

```
function c = ParabolaDR(exes,wys)
%Answers question 22 in Chapter 8. of Recktenwald.
V = vander(exes);
c = V\wys;
X = linspace(-10,10,100);

for k = 1:100
%evaluate using Horner's method
Y(k) = c(1);
for i = 2:3
   Y(k) =  Y(k)*X(k)  + c(i);
end
end

%Y = polyval(c,X);  This is the easy way to evaluate a polynomial in Matlab
plot(X,Y, exes,wys, '+');
```

I plotted all of the Parabolas. The one in (d) is a line, and (e) yields a singular system. For question 22 determine the condition numbers of the matrices used to obtain the interpolating parabola.

Here is the prettiest. It's (c).
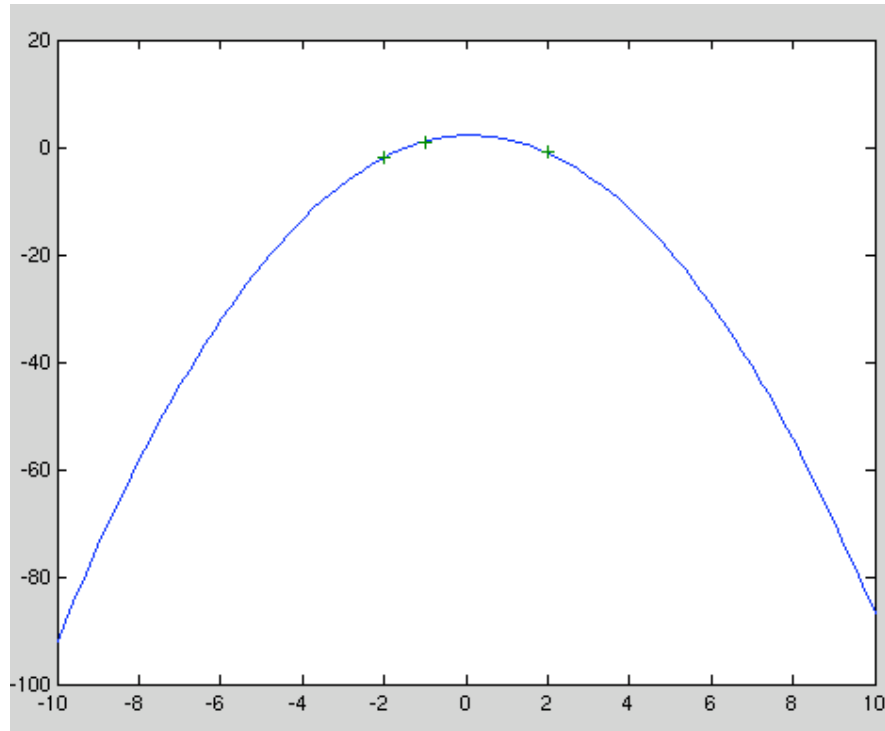
None of the condition numbers (except for the singular system) were very big due to the small problem size and small numbers.

8-30 The question has a bug. The condition number is one for any diagonal matrix where

all values on the main diagonal are the same. The is true using any p-norm.

8-31 To show that $\mathcal{K}(A) \geq 1$ for any A, we have

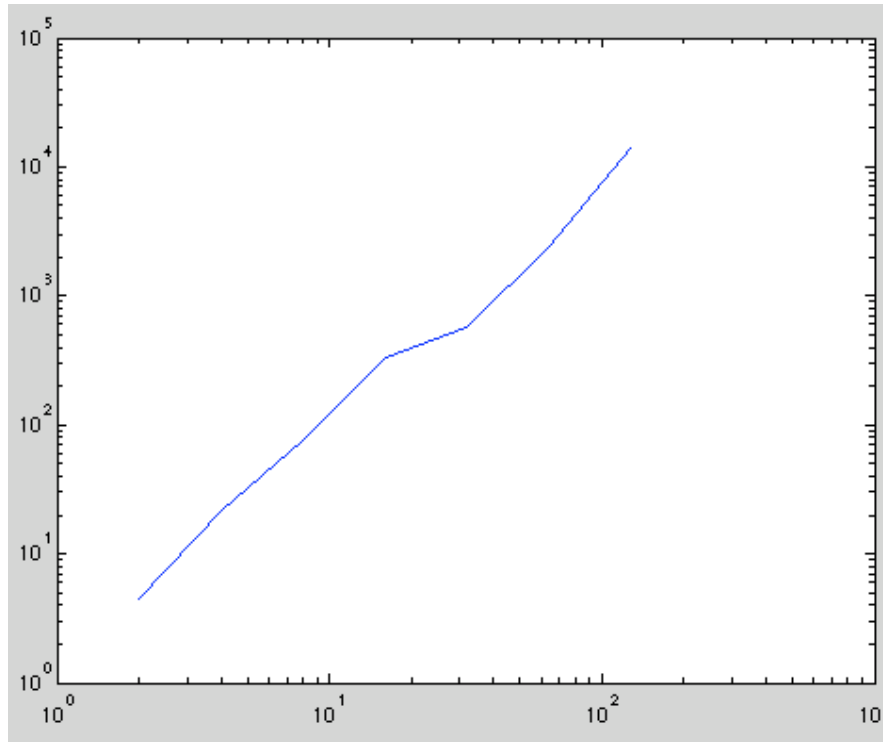$$\mathcal{K}(A) = ||A||\,||A^{-1}|| \geq ||AA^{-1}|| = ||I|| = 1$$

8-35 My matlab M-file:

```
function condsurveyDR
%Answers question 35 in Chapter 8. of REcktenwald.

for i = 1:7
    ave = 0; miny = realmax; maxy = 1;
    X(i) = 2^(i);
    for j = 1:12
      A = rand(X(i));
      C = cond(A,2);
      miny = min(C,miny);
      maxy = max(C,maxy);
      ave = ave + C/10;
    end
    %Throw out min and max value in an effort to smooth things out
    ave = ave - (miny/10 + maxy/10);
    Aave(i) = ave;
end

loglog(X,Aave);
```

And this is the plot I obtained:

Note: I use 12 samples and throw out the max and min. This seems to smooth things out a bit. For question 35 repeat the experiments using $\mathcal{K}_1(A)$ and $\mathcal{K}_\infty(A)$. Similar results are observed using $\mathcal{K}_1(A)$ and $\mathcal{K}_\infty(A)$. There is an increase in condition number proportional to the size of the random array.