CISC271
Fall 2005
Homework for week 10
in preparation for quiz 4
Solutions

This homework is about numerical quadrature..

Q1. I derived the relation E2 = (S2 -S1)/3 for the adaptive trapezoid rule. Do a similar derivation for Simpson's rule.

Solution: For Simpson's rule we have the error term as roughly $c|\frac{h^5}{90}|$, where c can be considered to be a constant over a small interval. Using the notation as in class we have

$$S1 + E1 = S2 + E2 \tag{1}$$

where we use E1 = $|\frac{h^5}{90}|$ and E2 = $|2\frac{(h/2)^5}{90}|$. Thus E2 = E1/16 so using equation (1) we have S1 + E1 = S2 + E1/16 which implies E2 = (S1-S2)/15.

Q2. What do the following lines of Matlab do? More to the point what is the value of Z?

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X = linspace(0,1)
Y = X.^2
Z = sum(Y)/100
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Solution: This almost works as a method that uses the trapezoid rule to approximate the integral

$$\int_0^1 x^2 dx$$

However, there are 99 panels not 100. So the width of each panel is 1/99. Furthermore, we have to divide Y(1) and Y(100) by 2. Since Y(1) 0^2 there is no need to divide it.

So Z*100/99 - 1/200 is the approximate area that the trapezoid rule would return. Nevertheless the actual area is 1/3. The original value for Z = 0.3350 and the trapezoid rule on 99 panels yields 0.3334.

The following questions are from Recktenwald Chapter 11.

Questions 1 and 5. asks you to evaluate to integrals by hand and then use the trapezoid (Simpson's rule) by hand and then use the supplied trapezoid (simpson) routines. In if you have access to the symbolic toolbox (this should be installed on the school's machines and it comes with the student version of Matlab) try integrating using the built in symbolic integrator *int*.

Solution:

Integrating exactly (i used Matlab int) I obtained the values:

$$I_1 = \int_{-\pi/2}^{\pi} (x^2 + x + 1)dx = 20.0408$$

and

$$I_2 = \int_{\sqrt{(3)}}^{-5} (x^3 - 1)dx = 160.7321$$

Then using the trapezoid and simpson functions I obtained the following approximations

Two panel trapezoid A1 = 24.4011 and A2 = 223.0477 Two panel simpson A1 = 20.0408 and A2 = 160.7321

Note: A one panel application of Simpson's rule would have obtained the same value for A1 and A2.

Questions 16 and 25 asks you to compare the performance of different numerical quadrature functions.

Solution:

$$I = \int_{0}^{1} (\sqrt{x})dx = .6667$$

and my experimental results are:

	2	4	8	16
trapezoid	0.6036	0.6433	0.6581	0.6636
simpson	0.6565	0.6631	0.6654	0.6662
gaussquad	0.6671	0.6668	0.6667	0.6667

Now using the adaptive algorithms

	0.1	0.01	0.001	0.0001
adaptsimpson	0.6565	0.6565	0.6631	0.6667
quad	0.6654	0.6654	0.6654	0.6662
quadl	0.6650	0.6650	0.6666	0.6666