CISC271

Practice Final Exam
Solutions

2005

These questions should give you some idea of the *type* of question I may ask on the final.

1. I have reproduced the definitions of the MATLAB constants realmin and realmax and eps.

   Built-in Variable: realmin The smallest normalized floating point number that is representable. The actual value is system-dependent. On machines that support 64-bit IEEE floating point arithmetic, 'realmin' is approximately 2.2251e-308

   - Built-in Variable: realmax The largest floating point number that is representable. The actual value is system-dependent. On machines that support 64-bit IEEE floating point arithmetic, 'realmax' is approximately 1.7977e+308

   Built-in Variable: eps The machine precision. More precisely, 'eps' is the largest relative spacing between any two adjacent numbers in the machine's floating point system. This number is obviously system-dependent. On machines that support 64 bit IEEE floating point arithmetic, 'eps' is approximately 2.2204e-16.

   (a) Is (realmin + eps) - realmin equal to zero? Explain.

   Solution: No, because realmin + eps $\neq$ realmin when using floating point arithmetic.

   (b) Is (realmax - eps) - realmax equal to zero? Explain.

   Solution: Yes, because realmax-eps = realmax when using floating point arithmetic.

2. The secant method for determining the root of a function can be implemented with the following formula:

$$x_{k+1} = x_k - f(x_k)\left[\frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}\right]$$

Explain how this formula may produce results that are completely unusable. (HINT: What happens to the denominator when $f(x_k)$ is approximately equal to $f(x_{k-1})$?)

Solution: When we subtract two values that are almost equal and divide we run the risk of overflow.

3. Using the secant algorithm and with initial values $a = 4$ and $b = 6$ what would be the next value obtained using the secant algorithm for the function:

$$x^2 - 25 = 0$$

Show your work.

Solution: Using the formula above we get: $6 - 11(2/20)$

4. Suppose that you are given the following experimental data which are from a function f(x):

x_i	1	2	3
y_i	2	4	2

Using Lagrange's formula find the unique polynomial that passes through all the points.

Solution: the Lagrange polynomials are:

$$L_1(x) = \frac{(x-2)(x-3)}{2}, L_2(x) = \frac{(x-1)(x-3)}{-1}, L_3(x) = \frac{(x-1)(x-2)}{2}$$

and the formula is:

$$P_2(x) = 2L_1(x) + 4L_2(x) + 2L_3(x)$$

5. (4) Consider the following matrix.

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

(a) Perform the calculations for Gaussian elimination by hand, to put the matrix B in triangular form. Show all of your work.

Solution:

Subtract row 1 from row 2, 3 ,and 4 to get

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 3 \end{pmatrix}$$

Now subtract row 2 from row 3 and 4 to get:

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Finally subtract row 3 from row 4 to get:

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) What is the LU decomposition of B.

Solution: $U = B$ above. Since the multipliers were all -1 we have

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

(c) What would the Matlab be to obtain an LU decomposition of B.

[L,U] = lu(B)

(d) Using the L_∞ matrix norm calculate the condition number of B. The inverse of B is

$$B^{-1} = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

Recalling that the ∞-norm is the max row sum norm we get

$$|B|_\infty = 40$$

6. Use Simpson's Rule to numerically estimate

$$\int_1^{2.718} \frac{1}{x} dx$$

The answer correct to 4 decimal places is 0.9999. What is the relative error of your computation.

Solution: Using a one panel Simpson's rule gives:

$$1.718/6(1/1 + 4/1.859 + 1/2.718) = 1.0078$$

The relative error is $|\frac{1.0078-0.9999}{0.9999}| = 0.0079$.

7. Evaluate $I = \int_0^1 x^2$ by a four point Guassian Quadrature formula. Just write out the formula, in terms of the points x_1, x_2, x_3, x_4 and weights w_1, w_2, w_3, w_4.

   Solution: We transform the problem to get an interval of integration [-1 ... 1] , apply the formula and post multiply by the scaling factor.

$$1/2(w_1(x_1/2 + 1/2)^2 + w_2(x_2/2 + 1/2)^2 + w_3(x_3/2 + 1/2)^2 + w_4(x_4/2 + 1/2)^2)$$

8. This question deals with least-squares approximations. Suppose that you were given many data points (e.g., a hundred) and that polynomials did not fit well. Plotting the data, you suspected that the y_i values were exponentially related to the x_i values. Show how'to fit data to the function

$$y = ce^{kx}$$

   that is, estimate values for c and k in the least squares sense.

   Solution: We take the natural log of both sides of the equation giving a linear relationship, that is: $\ln y = \ln c + kx$. Now call linefit and correct for the transformations as follows:

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ybar = log(y);
[a,b] = linefit(x,ybar);
k = a; c = exp(b);
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